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$N=2$ w_∞ supergravity

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We construct the gauge theory of $N=2$ w_∞ supergravity. The formulation presented here is obtained starting from a realization of the $N=2$ super-$\mathrm{W}_\infty(\lambda)$ algebra in terms of a supersymmetric BC system. We next apply a superbosonization of the BC superfields in terms of two real scalar superfields and take the classical limit $\hbar \to 0$. Different properties of the theory are discussed.

1. Introduction

W_N algebras are extensions of the Virasoro algebra which contain generators with all integer conformal spins $2 \leqslant s \leqslant N$ [1,2]. These algebras play an important role in the study of conformal field theories with $c \geqslant 1$ and have been studied from a variety of viewpoints [3,4].

A characteristic feature of the W_N algebras with $N \geqslant 3$ is that they are nonlinear, i.e., the commutator of two generators leads to polynomial expressions in the generators. These nonlinearities make the construction of a gauge theory of w_N gravity [#1] a nontrivial task. Some time ago the gauge theory of chiral [5,6] and nonchiral [7] w_3 gravity was constructed. More recently, w_3 gravity has been considered as a theory of critical W_3 strings [8].

In a separate development, algebras containing an infinite number of higher-spin generators have been discussed in the literature. The simplest example is the (linear) w_∞ algebra [9,10]. A gauge theory of w_∞ gravity was constructed in ref. [11]. It turns out that the w_∞ gravity theory can be consistently truncated to a w_N gravity theory for any N [11]. Upon this truncation the nonlinearities inherent to the w_N algebras are automatically reproduced. The quantum version of w_∞ is the W_∞ algebra of ref. [12]. Indeed, one can show that quantization deforms w_∞ gravity into W_∞ gravity [13].

Supersymmetry is clearly an element that could be added to the above picture. A supersymmetric version of the w_∞ algebra indeed exists [14,15]. However, this in itself does not imply that one can straightforwardly supersymmetrize the w_∞ gravity theory [11]. A w_3 supergravity theory has been constructed [6,16,17] but in contrast to the bosonic case the underlying algebra is nonuniversal, i.e., the structure constants depend on the explicit realization one is using for the currents generating the algebra.

In this letter we will show that there exists an $N=2$ supersymmetrization of w_∞ gravity. The underlying algebra is the (linear) $N=2$ super-w_∞ algebra [15] which is a contraction of the $N=2$ super-W_∞ algebra [18]. The theory is constructed by starting from a realization of the $N=2$ super-W_∞ algebra in terms of a supersymmetric

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[#1] The algebra w_N indicates the classical version of W_N. An algebra is called classical with respect to a given field realization if the algebra can be realized as a Poisson bracket algebra between currents which depend on the fields. The algebra is called quantum if, in order to realise the algebra, one needs to make more than single contractions between the currents (the single contractions correspond to the Poisson brackets).

BC system, as given in ref. [19]. Next the BC system is superbosonized [#2] in terms of two real scalar superfields. We will show that after this superbosonization one can define a classical limit of the field-theoretic representation, i.e., one can consistently throw away terms of higher order in $\hbar$. We thus obtain a field-theoretic realization of the $N=2$ super-w_∞ algebra.

2. The $N=2$ super-w_∞ algebra

The $N=2$ super-w_∞ algebra [15] is an extension of the $N=2$ super-Virasoro algebra which is defined by the following operator product (OPE) expansions:

$$w^{(1)}(1)w^{(1)}(2)\sim -2\frac{\theta_{12}w^{(3/2)}}{z_{12}},\quad w^{(3/2)}(1)w^{(1)}(2)\sim\left(\frac{\theta_{12}w^{(1)}}{z_{12}^2}-\frac{1}{2}\frac{\mathrm{D}_2w^{(1)}}{z_{12}}+\frac{\theta_{12}\,\partial_2w^{(1)}}{z_{12}}\right),$$

$$w^{(3/2)}(1)w^{(3/2)}(2)\sim\left(\frac{3}{2}\frac{\theta_{12}w^{(3/2)}}{z_{12}^2}-\frac{1}{2}\frac{\mathrm{D}_2w^{(3/2)}}{z_{12}}+\frac{\theta_{12}\,\partial_2w^{(3/2)}}{z_{12}}\right). \tag{1}$$

We use here a superfield notation where every superfield contains a bosonic and fermionic component field. We note that $w^{(1)}$ is commuting and $w^{(3/2)}$ is anticommuting. The superspace coordinates are $Z=(z,\theta)$. The superspace differential operator D is defined by $\mathrm{D}=\partial_\theta-\theta\partial$ with $\mathrm{D}^2=-\partial$. We have furthermore defined $\theta_{12}=\theta_1-\theta_2$ and $z_{12}=z_1-z_2+\theta_1\theta_2$. The notation $w^{(1)}(1)$ is a shorthand notation for $w^{(1)}(z_1,\theta_1)$. From eq. (1) one can recover the commutation relations of the generators of the algebra by multiplying the OPEs by the parameters of the corresponding transformations and integrating over the superspace coordinates.

To obtain the $N=2$ super-w_∞ algebra one extends the $N=2$ super-Virasoro algebra by additional generators $w^{(s)}$ with $s=2,\frac{5}{2},3,\dots$. The generators with integer (half-integer) s are commuting (anticommuting). The $N=2$ super-w_∞ algebra is then defined by the following OPE expansions. The OPE expansion of two generators $w^{(s)}$, $w^{(t)}$ where both s and t are half-integer or one is integer and the other half-integer is given by

$$w^{(s)}(1)w^{(t)}(2)\sim(-)^{|2s+1|_2}\left((s+t-\tfrac{3}{2})\frac{\theta_{12}w^{(s+t-3/2)}}{z_{12}^2}-\frac{1}{2}\frac{\mathrm{D}_2w^{(s+t-3/2)}}{z_{12}}+(s-\tfrac{1}{2})\frac{\theta_{12}\,\partial_2w^{(s+t-3/2)}}{z_{12}}\right), \tag{2}$$

where $|s|_2$ is equal to zero for s even and 1 for s odd. On the other hand, if both s and t are integer, the OPE expansion is given by

$$w^{(s)}(1)w^{(t)}(2)\sim -2\frac{\theta_{12}w^{(s+t-1/2)}}{z_{12}}. \tag{3}$$

The superfields $\{w^{(s)},w^{(s+1/2)}\}$ with s integer form $N=2$ multiplets with respect to the osp(2, 2) subalgebra of the $N=2$ super-Virasoro algebra. The osp(2, 2) subalgebra is defined by the $s=1,\frac{3}{2}$ transformations where the parameters $k^{(1)}$, $k^{(3/2)}$ which multiply the currents $w^{(1)}$, $w^{(3/2)}$ satisfy the conditions $\mathrm{D}^3k^{(1)}=\mathrm{D}^5k^{(3/2)}=0$.

It turns out that it is possible to extend further the $N=2$ super-w_∞ algebra with an additional $s=\frac{1}{2}$ generator $w^{(1/2)}$ with $w^{(1/2)}(1)w^{(1/2)}(2)\sim 0$. The OPE expansion of $w^{(1/2)}$ with $w^{(s)}$ (s integer) is given by

$$w^{(1/2)}(1)w^{(s)}(2)\sim\frac{w^{(s-1/2)}}{z_{12}}+\left((s-1)\frac{\theta_{12}w^{(s-1)}}{z_{12}^2}-\frac{1}{2}\frac{\mathrm{D}_2w^{(s-1)}}{z_{12}}\right). \tag{4}$$

For half-integer s, the OPE expansion is given by

[#2] The observation that bosonization might lead to a gauge theory of w_∞ supergravity was independently made by Sezgin [20].

$$w^{(1/2)}(1)w^{(s)}(2)\sim\left((s-1)\frac{\theta_{12}w^{(s-1)}}{z_{12}^2}-\frac{1}{2}\frac{D_2 w^{(s-1)}}{z_{12}}\right)$$

$$+\left(\tfrac{1}{2}(s-\tfrac{3}{2})\frac{w^{(s-3/2)}}{z_{12}^2}-\frac{1}{4}\frac{\theta_{12}D_2 w^{(s-3/2)}}{z_{12}^2}-\frac{1}{4}\frac{\partial_2 w^{(s-3/2)}}{z_{12}}\right). \tag{5}$$

3. Superbosonization

Our task is to find a field-theoretic representation of the $N=2$ super-w_∞ algebra. Our starting point for finding such a realization will be the $N=2$ super-W_∞ algebra [18]. We find it convenient to use here the formulation of ref. [19] where the algebra is called super-$W_\infty(\lambda)$. Here λ refers to a one-parameter choice of bases of the algebra. The $N=2$ super-$W_\infty(\lambda)$ algebra contains generators $W_\lambda^{(s)}$ with $s=\frac{1}{2}, 1, \frac{3}{2}, \ldots$. The terms of highest spin occurring in the OPE expansion of $W_\lambda^{(s)}(1)W_\lambda^{(t)}(2)$ exactly coincide with those of the $N=2$ super-w_∞ algebra given above. In addition the OPE expansions contain additional generators of lower spin. The $N=2$ super-$W_\infty(\lambda)$ algebra can be contracted to the $N=2$ super-w_∞ algebra [18]. In this contraction only the highest spin generators in the OPE expansion survive [#3]. One can view the contraction parameter as playing the role of $\hbar$. In this sense one can consider the $N=2$ super-w_∞ algebra as the classical limit of the quantum $N=2$ super-$W_\infty(\lambda)$ algebra. To summarize, schematically we have

$$W_\lambda^{(s)}(1)W_\lambda^{(t)}(2)\sim\text{as for the classical } N=2 \text{ super-}w_\infty \text{ algebra}+\hbar(\text{nonleading lower-spin generators}). \tag{6}$$

Our strategy is now the following. A realization of the $N=2$ super-$W_\infty(\lambda)$ algebra in terms of a supersymmetric BC system is known [19]. In order to be able to take the classical limit in this representation we first superbosonize the B, C superfields in terms of two real scalar superfields and then take the limit $\hbar\to 0$. Note that this procedure is the reverse of that of ref. [13]. We would like to stress that the $N=2$ super-W_∞ algebra is classical with respect to the B, C superfield realization [#4]. After the superbosonization the same algebra can be considered as a quantum algebra with respect to the two scalar superfields realization and only then it is possible to define a classical limit.

To be more explicit, we consider the following action [22]:

$$S=\frac{1}{\pi}\int d^2Z\, B\bar{D}C, \tag{7}$$

where B is a commuting superfield of weight λ and C is an anticommuting superfield of weight $\frac{1}{2}-\lambda$. The operator product of B, C is equal to

$$B(1)C(2)\sim\theta_{12}/z_{12}+\text{regular terms}. \tag{8}$$

A representation of the quantum $N=2$ super-$W_\infty(\lambda)$ algebra is then given by the OPEs of the following set of conserved currents:

$$W_\lambda^{(s)}=\sum_{i=0}^{2s-1}\tilde{A}^i(s,\lambda)(D^iB)(D^{2s-i-1}C), \tag{9}$$

with the coefficients $\tilde{A}^i(s,\lambda)$ given by [19]

[#3] An exception is the OPE expansion $W_\lambda^{(1/2)}(1)W_\lambda^{(s)}(2)$ where also the first nonleading spin generator survives.

[#4] the classical gauge theory corresponding to the $N=2$ super-W_∞ algebra has been given in ref. [21]. It is interesting in its own right to compare the quantum theory of the $N=2$ w_∞ supergravity theory constructed in this paper with the quantum theory of the $N=2$ W_∞ supergravity theory of ref. [21]. It is not obvious to us what the exact relationship between the two quantum theories is.

$$\tilde{A}^i(s,\lambda)=\frac{1+|2s|_2|i+1|_2}{1+|2s|_2}\frac{(-)^{[s]+[i/2]+|2s+1|_2|i+1|_2}}{(-[-s])_{[s]-|2s|_2|i|_2}}\binom{[s]-1+|2s|_2|i+1|_2}{[i/2]}$$
$$\times(2\lambda-[s])_{[i/2]+|2s+1|_2|i|_2}(-2\lambda-[s]+1)_{[s]-[i/2]-|i|_2}\,, \tag{10}$$

where $(a)_n\equiv(a+n-1)!/(a-1)!$ and $[a]$ denotes the integer part of a. In eq. (10) a normal ordering with respect to the modes of the currents is understood (see ref. [3] for more details on the normal ordering).

The BC system can be superbosonized in terms of two real scalar superfields ϕ, $\bar{\phi}$ as follows [23]:

$$B=\exp(\phi)\,,\quad C=\exp(-\phi)\,\mathrm{D}\bar{\phi}\,. \tag{11}$$

The basic operator product expansion of ϕ, $\bar{\phi}$ is given by

$$\phi(1)\bar{\phi}(2)\sim-\ln z_{12}\,. \tag{12}$$

From eqs. (11), (12) one can derive that the operator product expansion $B(1)C(2)$, including the regular terms, is given by

$$B(1)C(2)\sim\exp[\phi(1)-\phi(2)]\left(\frac{\theta_{12}}{z_{12}}+\mathrm{D}_2\bar{\phi}\right). \tag{13}$$

From this one may determine the superbosonized form of the currents $W^{(s)}_\lambda$ given in eq. (9). We next obtain a field-theoretic representation for the currents $w^{(s)}$ by taking the classical limit.

As an example we consider the first few currents:

$$W^{(1/2)}_\lambda=BC\,,\quad W^{(1)}_\lambda=(1-2\lambda)(\mathrm{D}B)C-2\lambda B(\mathrm{D}C)\,,$$
$$W^{(3/2)}_\lambda=\tfrac{1}{2}(1-2\lambda)(\partial B)C-\tfrac{1}{2}(\mathrm{D}B)(\mathrm{D}C)-\lambda B(\partial C)\,. \tag{14}$$

Their superbosonized form is given by [#5]

$$W^{(1/2)}_\lambda=\mathrm{D}\bar{\phi}\,,\quad W^{(1)}_\lambda=\mathrm{D}\phi\,\mathrm{D}\bar{\phi}+\sqrt{\hbar}\,\partial\phi+2\lambda\sqrt{\hbar}\,\partial\bar{\phi}\,,$$
$$W^{(3/2)}_\lambda=\tfrac{1}{2}\partial\phi\,\mathrm{D}\bar{\phi}+\tfrac{1}{2}\mathrm{D}\phi\,\partial\bar{\phi}+\tfrac{1}{2}\sqrt{\hbar}\,\partial\mathrm{D}\phi-\lambda\sqrt{\hbar}\,\partial\mathrm{D}\bar{\phi}\,. \tag{15}$$

The corresponding classical currents $w^{(s)}$ are given by

$$w^{(s)}=\lim_{\hbar\to0}W^{(s)}_\lambda \tag{16}$$

or

$$w^{(1/2)}=\mathrm{D}\bar{\phi}\,,\quad w^{(1)}=\mathrm{D}\phi\,\mathrm{D}\bar{\phi}\,,\quad w^{(3/2)}=\tfrac{1}{2}\partial\phi\,\mathrm{D}\bar{\phi}+\tfrac{1}{2}\mathrm{D}\phi\,\partial\bar{\phi}\,. \tag{17}$$

Note that the λ dependence disappears in the classical limit.

Since we are interested in the classical limit of the $W^{(s)}_\lambda$ currents we only need to determine the terms of highest power in ϕ, $\bar{\phi}$ in their expressions. Using this fact it is not too difficult to find a closed expression for the classical currents $w^{(s)}$. After some algebra we find

$$w^{(s)}=\sum_{i=0}^{2s-1}\tilde{A}^i(s,\lambda)\,\mathrm{D}^i_1\mathrm{D}^{2s-i-1}_2[\exp(\theta_{12}\mathrm{D}_2\phi+z_{12}\,\partial_2\phi)\,\mathrm{D}_2\bar{\phi}]\Big|_{z_{12}=\theta_{12}=0}\,. \tag{18}$$

Using the expression for the coefficients $\tilde{A}^i(s,\lambda)$ given in eq. (10) we find for half-integer s that

$$w^{(s)}=(\partial\phi)^{s-1/2}\,\mathrm{D}\bar{\phi}+\tfrac{1}{2}\mathrm{D}[\mathrm{D}\phi\,(\partial\phi)^{s-3/2}\,\mathrm{D}\bar{\phi}]\,. \tag{19}$$

[#5] In the next two equations we have indicated the explicit factors of $\hbar$. They can be easily recovered by dimension counting. The dimension of the ϕ, $\bar{\phi}$ superfields is $\sqrt{\hbar}$.

For integer s we find

$$w^{(s)}=\mathrm{D}\phi\,(\partial\phi)^{s-1}\,\mathrm{D}\bar{\phi}\,. \tag{20}$$

Here we have made use of the fact that for half-integers $s\geqslant\frac{3}{2}$ the following identities hold:

$$\sum_{i\ \mathrm{even}}^{2s-1}(-)^{i/2}\tilde{A}^i(s,\lambda)=+\tfrac{1}{2}\,,\quad \sum_{i\ \mathrm{odd}}^{2s-2}(-)^{i/2-1/2}\tilde{A}^i(s,\lambda)=-\tfrac{1}{2}\,. \tag{21}$$

Similarly, for integer s we have

$$\sum_{i=0}^{2s-1}(-)^{i+[i/2]}\tilde{A}^i(s,\lambda)=-1\,. \tag{22}$$

One can verify that taking single contractions between the currents $w^{(s)}$ (or, equivalently, by taking Poisson brackets) leads to the OPEs corresponding to the $N=2$ super-w_∞ algebra given in section 2. The currents $w^{(s)}$ form the basic ingredient in the construction of the gauge theory of $N=2$ w_∞ supergravity.

4. $N=2$ w_∞ supergravity

Our starting point is a free action for the scalar superfields ϕ, $\bar{\phi}$:

$$S_0=\int \mathrm{d}^2Z\,\mathrm{D}\phi\,\bar{\mathrm{D}}\bar{\phi}\,, \tag{23}$$

where $\bar{\mathrm{D}}=\partial_{\bar{\theta}}-\bar{\theta}\bar{\partial}$. This action is invariant under global $N=2$ super-w_∞ transformations with parameters $k_{(s)}$ satisfying $\bar{\mathrm{D}}k_{(s)}=0$. Under a spin-$s$ transformation the superfield ϕ transforms as follows:

$$\delta(k_{(s)})\phi(2)=\frac{1}{2\pi\mathrm{i}}\oint \mathrm{d}Z_1\,k_{(s)}(1)w^{(s)}(1)\phi(2)\,, \tag{24}$$

and similarly for $\bar{\phi}$. Note that $k_{(s)}$ is commuting (anticommuting) for half-integer (integer) s. Using the explicit form of the currents $w^{(s)}$ given in the previous section we find

$$\delta\phi=\sum_{s=3/2,5/2,\dots}^{\infty}[k_{(s)}(\partial\phi)^{s-1/2}-\tfrac{1}{2}\mathrm{D}k_{(s)}\,\mathrm{D}\phi\,(\partial\phi)^{s-3/2}]+\sum_{s=1,2,\dots}^{\infty}k_{(s)}\,\mathrm{D}\phi\,(\partial\phi)^{s-1}\,,$$

$$\delta\bar{\phi}=\sum_{s=3/2,5/2,\dots}^{\infty}\{-(s-\tfrac{1}{2})\,\mathrm{D}[k_{(s)}(\partial\phi)^{s-3/2}\,\mathrm{D}\bar{\phi}]+\tfrac{1}{2}\mathrm{D}k_{(s)}\,(\partial\phi)^{s-3/2}\,\mathrm{D}\bar{\phi}+\tfrac{1}{2}(s-\tfrac{3}{2})\,\mathrm{D}[\mathrm{D}k_{(s)}\,\mathrm{D}\phi(\partial\phi)^{s-5/2}\,\mathrm{D}\bar{\phi}]\}$$

$$+\sum_{s=1,2,\dots}^{\infty}[-k_{(s)}(\partial\phi)^{s-1}\,\mathrm{D}\bar{\phi}-(s-1)\,\mathrm{D}(k_{(s)}\,\mathrm{D}\phi\,(\partial\phi)^{s-2}\,\mathrm{D}\bar{\phi})]\,. \tag{25}$$

In this section we will not consider the $s=\frac{1}{2}$ transformation.

To gauge a chiral $N=2$ super-w_∞ symmetry, we allow the parameters $k_{(s)}$ to depend on Z as well as $\bar{Z}$, i.e., $\bar{\mathrm{D}}k_{(s)}\neq0$. We must also introduce gauge fields $A_{(s)}$ and add gauge field $\times$ current terms to the action:

$$S_{\mathrm{chiral}}=\int \mathrm{d}^2Z\left(\mathrm{D}\phi\,\bar{\mathrm{D}}\bar{\phi}+\sum_{s=1}^{\infty}A_{(s)}w^{(s)}\right). \tag{26}$$

The Noether procedure now goes as follows. The variation of ϕ, $\bar{\phi}$ in the kinetic term is cancelled by the leading-order transformation of the gauge field which is of the form $\delta A_{(s)}=\bar{\mathrm{D}}k_{(s)}+\dots$. We next vary the currents using the OPE expansion of the $N=2$ super-w_∞ algebra. This variation is cancelled by adding terms to the transformation rule of the gauge field $A_{(s)}$ so that its total variation $\delta A_{(s)}=\bar{D}k_{(s)}+\hat{\delta}A_{(s)}$ for s half-integer and t integer is given by

$$\delta A_{(s)}=\bar{D}k_{(s)}-2\sum_{t=1,2,\ldots}^{s-1/2}A_{(t)}k_{(s-t+1/2)}\,. \qquad (27)$$

In all other cases we have

$$\delta A_{(s)}=\bar{D}k_{(s)}+\left(\sum_{t=1,2,\ldots}^{s}+\sum_{t=3/2,5/2,\ldots}^{[s]+1/2}\right)[-(t-\tfrac{1}{2})A_{(t)}\,\partial k_{(s-t+3/2)}$$
$$+\tfrac{1}{2}(-)^{|2t|_2}\,\mathrm{D}A_{(t)}\,\mathrm{D}k_{(s-t+3/2)}+(s-t+1)\,\partial A_{(t)}k_{(s-t+3/2)}]\,. \qquad (28)$$

We next consider the nonchiral gauging. Following refs. [7,16] we introduce gauge fields $A_{(s)}$, $\bar{A}_{(s)}$ and currents $w^{(s)}$, $\bar{w}^{(s)}$ corresponding to the left- and right-handed symmetries (with parameters $k_{(s)}$, $\bar{k}_{(s)}$), and auxiliary fields F, $\bar{F}$, G, $\bar{G}$. We then find that the action for nonchiral $N=2$ super-w_∞ is given by

$$S_{\mathrm{nonchiral}}=\int \mathrm{d}^2Z\Bigg(-\mathrm{D}\phi\,\bar{\mathrm{D}}\bar{\phi}-F\bar{G}+\bar{F}G+F\,\bar{\mathrm{D}}\bar{\phi}+\mathrm{D}\phi\,\bar{G}-\bar{F}\,\mathrm{D}\bar{\phi}-\bar{\mathrm{D}}\phi\,G$$
$$+\sum_{s=1}^{\infty}[A_{(s)}w^{(s)}(F,G)+\bar{A}_{(s)}\bar{w}^{(s)}(\bar{F},\bar{G})]\Bigg)\,. \qquad (29)$$

The notation $w^{(s)}(F,G)(\bar{w}^{(s)}(\bar{F},\bar{G}))$ indicates that in the expression for $w^{(s)}$ ($\bar{w}^{(s)}$) everywhere $\mathrm{D}\phi$, $\mathrm{D}\bar{\phi}$ ($\bar{\mathrm{D}}\phi$, $\bar{\mathrm{D}}\bar{\phi}$) has been replaced by F, G ($\bar{F}$, $\bar{G}$). The action is invariant under the following nonchiral symmetries:

$$\delta\phi=\delta\phi(F)+\delta\phi(\bar{F})\,,\quad \delta\bar{\phi}=\delta\bar{\phi}(F,G)+\delta\bar{\phi}(\bar{F},\bar{G})\,,$$
$$\delta F=\mathrm{D}\delta\phi(F)\,,\quad \delta\bar{F}=\bar{\mathrm{D}}\delta\phi(\bar{F})\,,\quad \delta G=\mathrm{D}\delta\bar{\phi}(F,G)\,,\quad \delta\bar{G}=\bar{\mathrm{D}}\delta\bar{\phi}(\bar{F},\bar{G})\,,$$
$$\delta A_{(s)}=\bar{\mathrm{D}}k_{(s)}+\hat{\delta}A_{(s)}\,,\quad \delta\bar{A}_{(s)}=\mathrm{D}\bar{k}_{(s)}+\hat{\delta}\bar{A}_{(s)}\,. \qquad (30)$$

We have used here an obvious notation where, e.g., $\delta\phi(F)$ indicates that in the expression for $\delta\phi$ everywhere $\mathrm{D}\phi$ has been replaced by F, etc. Furthermore $\delta\phi(\bar{F})$ is obtained from $\delta\phi(F)$ by the replacements $k_{(s)}$, $F\to\bar{k}_{(s)}$, $\bar{F}$, and similarly for $\delta\bar{\phi}(\bar{F},\bar{G})$. Finally, $\hat{\delta}\bar{A}_{(s)}$ is obtained from $\hat{\delta}A_{(s)}$ by the replacements $k_{(s)}$, $A_{(s)}\to\bar{k}_{(s)}$, $\bar{A}_{(s)}$.

5. Discussion

In this letter we have constructed the gauge theory of $N=2$ w_∞ supergravity. Clearly, there are a number of directions in which our work can be further developed. First of all, it would be interesting to investigate whether the final result can be reformulated in a supergeometrical framework, along the lines discussed in ref. [23].

It would also be interesting to investigate whether the $N=2$ w_∞ supergravity theory allows a truncation with $N=1$ supersymmetry. For the (quantum) supersymmetric BC system such a truncation can be achieved by imposing the condition $C=\mathrm{D}B$ (for $\lambda=0$). It is not obvious whether a similar truncation is possible in the classical case. One way to investigate this would be to first perform the truncation $C=\mathrm{D}B$ in the quantum case and then try to superbosonize the B superfield. Since the B superfield contains only one real fermion, it seems that one would need a chiral bosonization.

Finally, another application of our result might be the construction of gauge theories of w_N supergravity theories with an underlying universal superalgebra. For this to work, it would be necessary to generalize the Stueckelberg symmetries of ref. [11]. At first sight it looks that the $N=2$ w_∞ supergravity theory has no

Stueckelberg symmetries at all. The reason for this is that the currents are nonlinear in the superfield ϕ but linear in $\bar{\phi}$. Therefore, at the classical level, one can never write the higher-spin currents as products of lower-spin currents. However, we should note the following. Although the supersymmetric BC system has only one $s=\frac{1}{2}$ current, for the ϕ, $\bar{\phi}$ system, one can construct (in the left-moving sector) two such currents, namely $w^{(1/2)}=\mathrm{D}\bar{\phi}$ as well as $\mathrm{D}\phi$. Using this second $s=\frac{1}{2}$ current a Sugawara construction of the $s=1$, $\frac{3}{2}$ supercurrents can be given [24]. In this context, it is also worth mentioning that it seems possible to express the quantum currents of the supersymmetric BC system in terms of (products of) lower-spin currents. It would be interesting to pursue this direction further, since it might lead to a proper definition of "W_N supergravity", and, possibly, "W_N superstrings".

After the completion of this work we received a preprint [25] in which a similar bosonization technique is used to arrive at two-scalar realizations of classical w_∞.

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